BRIEF COMMUNICATIONS

A CYLINDRICAL WIRE HEATED BY A CURRENT

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ABSTRACT: An examination is made of the radial distribution of temperature and current density in an infinitely long cylindrical wire, heated by a current allowing for the dependence of thermal and electrical conductivities on temperature.

The temperature distribution in an infinitely long cylindrical wire heated by a constant current, for constant values of the thermal conductivity λ and electrical conductivity σ has been given in [1], and for the case λ = const, σ = var—in [2].

This paper presents an approximate method of solution for the case $\lambda = \lambda(T)$.

The equation of energy balance for a cylindrical wire of unit length in the absence of axial overflow of heat may be written as

$$\sigma E^2 + \frac{1}{r}\frac{d}{dr}\left(r\lambda\frac{dT}{dr}\right) = 0, \tag{1}$$

where E is the longitudinal intensity of the electric field; r is the radius; T is the temperature. Using the heat conduction function $S = \int_0^T \lambda\, dT$ [3] and the relative radius $\rho = (r)/R$ (R being the wire radius), Eq. (1) may be transformed to the form

$$\sigma E^2 R^2 + \frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{dS}{d\rho}\right) = 0. \tag{2}$$

In reducing the material function $\sigma(S)$, i.e., the dependence of the electrical conductivity σ on the thermal conductivity function S, we obtain an equation with a single parameter, considering the electric field intensity E to be constant along the wire radius. The material function $\sigma(S)$ may be represented graphically, using the dependences $\sigma(T)$ and $S(T)$. A solution is effected by linearization of $\sigma(S)$ along the radius of the channel of the wire of the form $\sigma = AS + B$ (the simplest case). Then the solution of (2) for $A > 0$ becomes

$$S(\rho) = c_1 J_0(\nu) + c_2 Y_0(\nu) - B/A, \tag{3}$$

and for $A < 0$

$$S(\rho) = c_1 I_0(\nu) + c_2 K_0(\nu) - B/A, \tag{4}$$

where $\nu = \rho ER A^{1/2}$; $J_0(\nu)$ and $Y_0(\nu)$ are Bessel functions of zero order of the first and second kinds; $I_0(\nu)$ and $K_0(\nu)$ are modified Bessel function of zero order of the first and second kinds; c_1, c_2, are constants of integration.

Using the boundary conditions

$$\text{for } \rho = 1 \quad S = S_W \text{ and } q = -\frac{1}{R}\left(\frac{dS}{d\rho}\right)_{\rho=1}, \tag{5}$$

where S_W is the value of the thermal conductivity function along the wire surface; and q is the heat flux, we obtain the radial distribution of the heat conduction function for the case $A < 0$ (for metals):

$$S(\rho) = \frac{qR}{\nu_1 I_1(\nu_1)} I_0(\nu) - \frac{B}{A}. \tag{6}$$

Here $\nu_1 = ER(A)^{1/2}$ is determined from solution of the transcendental equation

$$\frac{\nu_1 I_1(\nu_1)}{I_0(\nu_1)} = \frac{qR}{S_{CT} + B/A}, \tag{7}$$

where the quantities q, R, A, B and S_W are assumed to be given, and $I_1(\nu_1)$ is a modified Bessel function of the first order and the first kind.

Thereafter we may determine the value E, and knowing the heat flux q and E, find the current. The radial distribution of current density (the density of the internal source of heat generation) may be expressed as

$$j(\rho) = \frac{\sigma_W E}{I_0(\nu_1)} I_0(\nu), \tag{8}$$

where σ_W is the electrical conductivity at the temperature of the wire surface.

Expression (8) illustrates the phenomenon of current-density "thermal skin effect" due to decrease of the electrical conductivity with increase of temperature. The radial distribution of temperature may be obtained from the profile $S(\rho)$, using the graphical dependence of $S(T)$.

If we use the boundary conditions

$$\text{for } \rho = 1 \quad S = S_W,$$
$$\text{for } \rho = 0 \quad S = S_0, \tag{9}$$

we obtain the following expressions for the distribution of the heat conduction function and of the specific heat flux through the wire surface,

$$S(\rho) = (S_0 + B/A) I_0(\nu) - B/A, \tag{10}$$

$$q = \sigma_0 \nu_1 I_1(\nu_1)/RA, \tag{11}$$

where σ_0 is the electrical conductivity at the temperature on the axis, and $\nu_1 = ER(A)^{1/2}$ is determined from (10) by substitution of the boundary conditions.

The accuracy of the approximate solution will depend on the nature of the approximation $\sigma(S)$. For increased accuracy of solution the dependence $\sigma(S)$ may be approximated by several straight lines [4].

It follows from the foregoing examination that for given temperatures on the axis and on the wire surface the required electrical power E does not depend on the wire radius. Then we will find constant temperature profiles with respect to the relative radius. The condition of similarity in this case is written in a similar way to the condition for a positive arc column [4]: ER = const for EI = const (i.e., with constant boundary conditions), where I is the current.

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3 May 1966

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